2.1 Chain Rule: Quiz B Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_quiz.py`). All problems are original; provenance is in `PROVENANCE_QB.md`. To the extent possible under law, the author(s) have waived all copyright and related or neighboring rights to this key. CC0 1.0 Universal: `https://creativecommons.org/publicdomain/zero/1.0/`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit, except where a row says otherwise.
- **Any valid method.** Expanding first, rewriting a quotient with a negative exponent, the quotient rule, the product rule, simplifying an identity first, or carrying the chain rule all the way through are all accepted; no method is required, and long but correct work gets full credit.
- **Notation.** $f'(x)$, y', $\frac{dy}{dx}$ and $\frac{d}{dx}[\ ]$ are all accepted in work. The key writes the chain rule as in the video: $\frac{d}{dx}[A(B(x))] \leftrightarrow \left(\frac{d}{ds}[A(s)]\right)\Big|_{s=B(x)} \cdot \frac{d}{dx}[B(x)]$, with the evaluation bar applied only after the $\frac{d}{ds}$ is gone.
- **Value vs. derivative.** A function value where a derivative was asked (or the reverse) earns 0 for that part.
- **Simplifying.** Answers need not be simplified; equivalent forms are correct.
- **Excluded values.** Where a formula or a method fails at some values of the variable, the key names them. No points are deducted for omitting statements of excluded values.

Quiz B

Item	Answer	Rubric / trap
1(a)	$3(2x^3 - x + 5)^2(6x^2 - 1)$	no earlier rule matches the whole power (its base is not x), so the chain rule with $A(s) = s^3$, $B(x) = 2x^3 - x + 5$: $\left(3s^2\right)\big\|_{s=2x^3-x+5} \cdot (6x^2 - 1)$. Expanding first also works. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* power-rule bait: $3(2x^3 - x + 5)^2$ alone drops $B'(x) = 6x^2 - 1$ and earns 0.
1(b)	$(5 - 3x^2)\cos(5x - x^3)$	chain rule, $A(s) = \sin s$, $B(x) = 5x - x^3$: $(\cos s)\big\|_{s=5x-x^3} \cdot (5 - 3x^2)$. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* $\cos(5x - x^3)$ alone drops $B'(x)$; $-\cos$ (or $-(5 - 3x^2)\cos$) is a Sine Rule sign slip; both earn 0.
1(c)	$4x^3 \sec(x^4)\tan(x^4)$	chain rule, $A(s) = \sec s$, $B(x) = x^4$: $(\sec s \tan s)\big\|_{s=x^4} \cdot 4x^3$. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* exponent placement: the outer function is the secant, not a fourth power; $4\sec^3 x \sec x \tan x$-type answers, or $\sec(x^4)\tan(x^4)$ alone, earn 0.
1(d)	$\dfrac{2x + 5}{3(x^2 + 5x)^{2/3}}$	rewrite as $(x^2 + 5x)^{1/3}$ first; chain rule, $A(s) = s^{1/3}$, $B(x) = x^2 + 5x$: $\left(\frac{1}{3}s^{-2/3}\right)\big\|_{s=x^2+5x} \cdot (2x + 5)$. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* power-rule bait on the radical: $\frac{1}{3}(x^2 + 5x)^{-2/3}$ alone earns 0.

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1(e)	$2t(3t+1)^4 + 12t^2(3t+1)^3$	product rule; inside it, $\frac{d}{dt}\left[(3t+1)^4\right] = \left(4s^3\right)\big\|_{s=3t+1} \cdot 3 = 12(3t+1)^3$. So $2t(3t+1)^4 + t^2 \cdot 12(3t+1)^3$, equivalently $2t(3t+1)^3(9t+1)$. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* $4t^2(3t+1)^3$ in the second term drops the inner 3; multiplying derivatives, $2t \cdot 12(3t+1)^3$, earns 0.
1(f)	$\dfrac{36}{(1-3x)^4}$	rewrite as $4(1-3x)^{-3}$; constant multiple, then chain rule, $A(s) = s^{-3}$, $B(x) = 1-3x$: $4 \cdot \left(-3s^{-4}\right)\big\|_{s=1-3x} \cdot (-3)$. The quotient rule also works. 1 pt correct outer derivative with the right rule(s), 1 pt the complete derivative. *Trap:* $B'(x) = -3$: dropping it gives $-\dfrac{12}{(1-3x)^4}$ (wrong sign and size).
2(a)	0	$\sec^2(3x) - \tan^2(3x) = 1$, so $y = 3^4 = 81$ for every x where it is defined. The chain-rule route ends in $4(\ldots)^3 \cdot \left[6\sec^2(3x)\tan(3x) - 6\tan(3x)\sec^2(3x)\right] = 0$. 2 pts, all or nothing for the value 0; work by either route. *Trap:* a constant in disguise; a nonzero answer comes from a slip in the chain-rule route, most often a dropped inner 3 or the derivative of $\sec(3x)$.
2(b)	0	$(t-2)(t+2) - t^2 + 12 = t^2 - 4 - t^2 + 12 = 8$, so $m(t) = 2$ for every t. The chain-rule route ends in $\frac{1}{3}(8)^{-2/3} \cdot [2t - 2t] = 0$. 2 pts, all or nothing for the value 0; work by either route. *Trap:* a constant in disguise; any t left in the answer earns 0.
2(c)	$4\left(\tan(2x) + x^6 - 3\sqrt[3]{x}\right)^3$	the derivative is with respect to $\tan(2x)+x^6-3\sqrt[3]{x}$, not x. Let $s = \tan(2x)+x^6-3\sqrt[3]{x}$; the bracket is s^4, so the derivative is $\left(4s^3\right)\big\|_{s=\tan(2x)+x^6-3\sqrt[3]{x}}$. Dividing $\frac{d}{dx}[\ldots]$ by $\frac{d}{dx}\left[\tan(2x) + x^6 - 3\sqrt[3]{x}\right]$ also works wherever that inner derivative exists and is nonzero (the substitution route has no such gap). Excluded values: x where $2x$ is an odd multiple of $\frac{\pi}{2}$; the division route also excludes $x = 0$ and every x where the inner derivative is 0 (infinitely many, since its trig term takes every large value on each period). 2 pts, all or nothing; work by either route. *Trap:* differentiating with respect to x: $4(\ldots)^3\left(2\sec^2(2x) + 6x^5 - x^{-2/3}\right)$ earns 0.
3(a)	1	$h(x) = \left(\sqrt[5]{x-1} + 2 - 2\right)^5 + 1 = (x-1) + 1 = x$, so $h'(x) = 1$ everywhere. Chain route: $g(2) = 3$, $f'(x) = 5(x-2)^4$, $g'(x) = \frac{1}{5}(x-1)^{-4/5}$, so $h'(2) = f'(3)\,g'(2) = 5 \cdot \frac{1}{5}$. 2 pts, all or nothing. *Trap:* not noticing $h(x) = x$ costs time, not points; $f'(2) = 0$ or $f'(2)\,g'(2) = 0$ earns 0.
3(b)	1	$k(x) = \sqrt[5]{(x-2)^5 + 1 - 1} + 2 = (x-2) + 2 = x$, so $k'(x) = 1$ for every x. Chain route: $g'(f(x))\,f'(x) = \frac{1}{5}\left[(x-2)^5\right]^{-4/5} \cdot 5(x-2)^4 = 1$ (for $x \neq 2$; the identity covers $x = 2$). 2 pts. *Trap:* an unsimplified chain-rule expression equivalent to 1 earns full credit; $g'(x)f'(x)$ earns 0.
4(a)	$8x^3\dfrac{dx}{dy} + 3y^2$	constant multiple, then the chain rule on x^4 with $A(s) = s^4$ and $B(y) = x$: $2 \cdot \left(4s^3\right)\big\|_{s=x} \cdot \dfrac{dx}{dy}$; $\dfrac{d}{dy}[y^3] = 3y^2$ by the Power Rule. 2 pts as the trap note says. *Trap:* treating x as the variable: $8x^3$ (with y^3 dropped as a constant) earns 0; $8x^3 + 3y^2$ earns 1; $8x^3\dfrac{dx}{dy}$ alone earns 1.
4(b)	$3x^2\sec^2(x^3)\dfrac{dx}{dy}$	two chain layers: $A(s) = \tan s$, $B(y) = x^3$, and $\dfrac{d}{dy}[x^3] = 3x^2\dfrac{dx}{dy}$: $\left(\sec^2 s\right)\big\|_{s=x^3} \cdot 3x^2\dfrac{dx}{dy}$. 2 pts as the trap note says. *Trap:* $3x^2\sec^2(x^3)$ (no $\dfrac{dx}{dy}$) earns 0; $\sec^2(x^3)\dfrac{dx}{dy}$ earns 1.
5(a)	-2	$g(3) = -2$, $g'(3) = -1$ (slope of the segment containing $x = 3$); $f'(-2) = 2$. $h'(3) = f'(g(3))\,g'(3) = (2)(-1)$. 1 pt inner value and slope, 1 pt outer slope at the right x, 1 pt value. *Trap:* $f'(3)\,g'(3) = -1$ (outer slope read at 3 instead of at $g(3) = -2$); $f'(-2) = 2$ alone (inner slope dropped).

Item	Answer	Rubric / trap
5(b)	4	$f(-3) = -1$, $f'(-3) = 2$ (slope of the segment containing $x = -3$); $g'(-1) = 2$. $k'(-3) = g'(f(-3))\, f'(-3) = (2)(2)$. 1 pt inner value and slope, 1 pt outer slope at the right x, 1 pt value. *Trap:* $g'(-3)\, f'(-3) = 2$ (outer slope read at -3 instead of at $f(-3) = -1$); $g'(-1) = 2$ alone (inner slope dropped).
6(a)	9	$h'(2) = g'(f(2))\, f'(2) = g'(0) \cdot 3 = (3)(3)$. 1 pt $f(2) = 0$, 1 pt $g'(0)$ and $f'(2)$, 1 pt value. *Trap:* $g'(2)\, f'(2) = -6$ reads the outer slope in the wrong row. Columns run x, $f(x)$, $g'(x)$, $f'(x)$, $g(x)$.
6(b)	20	$k'(2) = f'(2^2) \cdot 2(2) = f'(4) \cdot 4 = (5)(4)$. 1 pt chain rule with inner t^2, 1 pt entries, 1 pt value. *Trap:* $f'(4) = 5$ alone drops the inner $2t$; $f'(2) \cdot 4 = 12$ reads the outer slope at t instead of t^2.
6(c)	2	$h(2) = g(f(2)) = g(0)$. 2 pts, all or nothing. *Trap:* this part asks for a value of h, not h': 9 earns 0; $g(2) = 4$ earns 0.

Points: $12 + 6 + 4 + 4 + 6 + 8 = 40$.